



ELEMENTARY **2ND EDITION**
Algebra



Graphs and Authentic Applications



JAY LEHMANN

Prepare for Exams with Lehmann's New Interactive Video Lecture Series

PEARSON ALWAYS LEARNING

Finding Equations of Linear Models [more info](#)

Objective 1

Objective 2

form of an equation that models the Median age of the U.S. population x years after 1970. Use this model to predict the median age in 2020.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

01:10 / 06:51

Playback trouble?

PEARSON ALWAYS LEARNING

Finding Equations of Linear Models [more info](#)

Use the data points $(10, 30.0)$ and $(20, 32.8)$ to write the slope-intercept form of an equation that models the Median age of the U.S. population x years after 1970. Use this model to predict the median age in 2020.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

01:10 / 06:51

Playback trouble?



Students can make the most of their study time by preparing for exams with the new Interactive Lecture Series.

Interactive Lectures highlight key examples and exercises from every section of the textbook. A new interface allows easy navigation to sections, objectives, and examples. These videos are available in [MyMathLab](#).

Jay Says...

Before writing my algebra series, it was apparent that my students couldn't relate to the applications in the course; they would repeatedly ask, "What is this good for?" To try to bridge that gap, I wrote some labs, which helped my students collect data, find models via curve fitting, and use the models to make estimates and predictions. My students really loved working with the current, compelling, and authentic data and experiencing how mathematics truly is useful.

My students' response was so strong that I decided to write an algebra series. Little did I know that, to realize this goal, I'd need to embark on a 15-year challenging journey, but the rewards of hearing such excitement from students and faculty across the country have made it all worthwhile! I'm proud to have played even a small role in raising people's respect and enthusiasm for mathematics.

I've tried to honor my inspiration: By working with authentic data, students can experience the power of mathematics. A random-sample study at my college suggests that I'm achieving this goal. The study concludes that students who used my series were more likely to feel that mathematics would be useful in their lives ($p = 0.0061$) as well as in their careers ($p = 0.024$).

In addition to curve fitting, my approach includes other types of meaningful modeling, directed-discovery explorations, conceptual questions, and, of course, a large bank of skill problems. The curve-fitting applications serve as a portal for students to see the usefulness of mathematics so that they become fully engaged in the class. Once involved, they're more receptive to all aspects of the course.

Elementary Algebra

Graphs & Authentic Applications

Second Edition

Jay Lehmann

College of San Mateo

PEARSON

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montréal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Editorial Director: Chris Hoag
Editor in Chief: Michael Hirsch
Senior Acquisitions Editor: Dawn Giovanniello
Senior Content Editor: Lauren Morse
Editorial Assistant: Ashley Yee
Senior Managing Editor: Karen Wernholm
Associate Managing Editor: Tamela Ambush
Digital Assets Manager: Marianne Groth
Media Producer: Shana Siegmund
QA Manager, Assessment Content: Marty Wright
Executive Content Manager: Rebecca Williams
Senior Content Developer: John Flanagan
Executive Marketing Manager: Michelle Renda
Associate Marketing Manager: Alicia Frankel
Marketing Assistant: Susan Mai
Liaison Manager, Text Permissions Group: Joseph Croscup
Procurement Specialist: Debbie Rossi
Associate Director of Design, USHE North and West: Andrea Nix
Program Design Lead & Text Design: Beth Paquin
Production Coordination and Composition: Integra
Illustrations: Electronic Publishing Services Inc.
Cover Design: StudioWink
Cover Image: David Caudery/Total Guitar Magazine via Getty Images

Many of the designations used by manufacturers and sellers to distinguish their products are claimed as trademarks. Where those designations appear in this book, and Pearson Education was aware of a trademark claim, the designations have been printed in initial caps or all caps.

Library of Congress Cataloging-in-Publication Data

Lehmann, Jay.

Elementary algebra : graphs & authentic applications / Jay Lehmann,
College of San Mateo.—Second edition.

pages cm

Summary: “Elementary algebra textbook organized around a curve fitting approach, with group explorations and an immediate introduction to modeling and authentic applications”—Provided by publisher.

Includes bibliographical references and index.

ISBN-13: 978-0-321-86827-5 (student edition)

ISBN-10: 0-321-86827-7 (student edition)

1. Algebra—Textbooks. I. Title.

QA152.3.L45 2015

512.9—dc23

2013000233

Copyright © 2015, 2008 Pearson Education, Inc.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America. For information on obtaining permission for use of material in this work, please submit a written request to Pearson Education, Inc., Rights and Contracts Department, 501 Boylston Street, Suite 900, Boston, MA 02116, fax your request to 617-671-3447, or e-mail at <http://www.pearsoned.com/legal/permissions.htm>.

1 2 3 4 5 6 7 8 9 10—CRK—17 16 15 14 13

PEARSON

ISBN-10: 0-321-86827-7

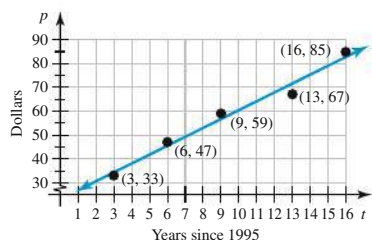
ISBN-13: 978-0-321-86827-5

**To Dylan, for your curiosity
and wonder of mathematics.**

Contents

PREFACE ix
TO THE STUDENT xiii
ACKNOWLEDGMENTS xiv
INDEX OF APPLICATIONS xvii

Average Ticket Prices for
Top-50-Grossing Concert Tours
(pp. 36–38)



Presidential Election Voter
Turnout (p. 86)

Year	Percent of Eligible Voters Who Voted
1980	59.2
1984	59.9
1988	57.4
1992	61.9
1996	54.2
2000	54.7
2004	63.8
2008	63.6
2012	57.5

Percentages of Adult Internet
Users Who Use Social
Networking Sites (p.165)

Year	Percent
2005	8
2006	16
2008	29
2009	46
2010	61
2011	65

1 INTRODUCTION TO MODELING 1

1.1 Variables and Constants 1
1.2 Scattergrams 13
1.3 Exact Linear Relationships 25
1.4 Approximate Linear Relationships 36
Taking it to the Lab: Climate Change Lab • Volume Lab • Linear
Graphing Lab: Topic of Your Choice 46
CHAPTER SUMMARY 48
Key Points of Chapter 1 48
Chapter 1 Review Exercises 49
Chapter 1 Test 52

2 OPERATIONS AND EXPRESSIONS 53

2.1 Expressions 53
2.2 Operations with Fractions 61
2.3 Adding Real Numbers 71
2.4 Change in a Quantity and Subtracting Real Numbers 79
2.5 Ratios, Percents, and Multiplying and Dividing Real Numbers 89
2.6 Exponents and Order of Operations 100
Taking it to the Lab: Climate Change Lab • Stocks Lab 108
CHAPTER SUMMARY 111
Key Points of Chapter 2 111
Chapter 2 Review Exercises 113
Chapter 2 Test 114
Cumulative Review of Chapters 1 and 2 115

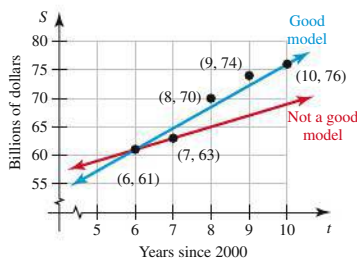
3 USING SLOPE TO GRAPH LINEAR EQUATIONS 117

3.1 Graphing Equations of the Form $y = mx + b$ 117
3.2 Graphing Linear Models; Unit Analysis 125
3.3 Slope of a Line 134
3.4 Using Slope to Graph Linear Equations 143
3.5 Rate of Change 154
Taking it to the Lab: Climate Change Lab • Workout
Lab • Balloon Lab 168
CHAPTER SUMMARY 171
Key Points of Chapter 3 171

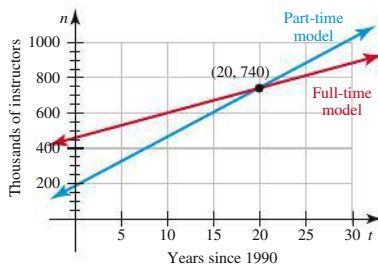
College Freshmen Whose Average Grade in High School Was an A (p. 214)

Year	Percent
1970	19.6
1980	26.6
1985	28.7
1990	29.4
1995	36.1
2000	42.9
2005	46.6
2010	48.4

Total College Student Discretionary Spending (pp. 256–257)



Numbers of Part-Time and Full-Time Instructors (p. 294)



4

SIMPLIFYING EXPRESSIONS AND SOLVING EQUATIONS 177

- 4.1 Commutative, Associative, and Distributive Laws 177
- 4.2 Simplifying Expressions 187
- 4.3 Solving Linear Equations in One Variable 192
- 4.4 Solving More Linear Equations in One Variable 203
- 4.5 Comparing Expressions and Equations 217
- 4.6 Formulas 224
- CHAPTER SUMMARY 234
- Key Points of Chapter 4 234
- Chapter 4 Review Exercises 236
- Chapter 4 Test 238
- Cumulative Review of Chapters 1–4 238

5

LINEAR EQUATIONS IN TWO VARIABLES 240

- 5.1 Graphing Linear Equations 240
- 5.2 Finding Linear Equations 247
- 5.3 Finding Equations of Linear Models 255
- 5.4 Using Linear Equations to Make Estimates and Predictions 264
- 5.5 Solving Linear Inequalities in One Variable 272
- Taking it to the Lab: Climate Change Lab • Rope Lab • Shadow Lab • Linear Equation Lab: Topic of Your Choice 282
- CHAPTER SUMMARY 285
- Key Points of Chapter 5 285
- Chapter 5 Review Exercises 287
- Chapter 5 Test 288

6

SYSTEMS OF LINEAR EQUATIONS 290

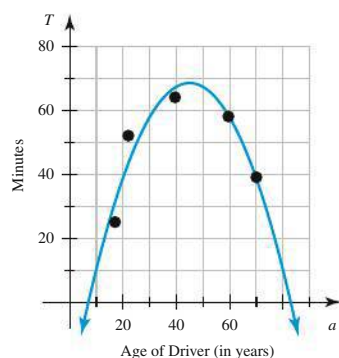
- 6.1 Using Graphs and Tables to Solve Systems 290
- 6.2 Using Substitution to Solve Systems 300
- 6.3 Using Elimination to Solve Systems 306
- 6.4 Using Systems to Model Data 313
- 6.5 Perimeter, Value, Interest, and Mixture Problems 320
- 6.6 Linear Inequalities in Two Variables; Systems of Linear Inequalities in Two Variables 331
- Taking it to the Lab: Climate Change Lab 340
- CHAPTER SUMMARY 341
- Key Points of Chapter 6 341
- Chapter 6 Review Exercises 343
- Chapter 6 Test 345
- Cumulative Review of Chapters 1–6 346

7

POLYNOMIALS 349

- 7.1 Graphing Quadratic Equations 349
- 7.2 Quadratic Models 355

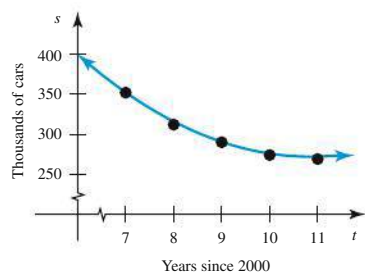
Average Times Spent Driving Each Day (pp. 355–356)



Worldwide iPhone Sales (p. 456–457)

Year	Sales (millions)
2007	1.4
2008	11.6
2009	20.7
2010	40.0
2011	72.3

Sales of Gasoline–Electric Hybrid Cars (pp. 499–501)



Households That Use Online Banking (pp. 547–548)

Year	Households That Use Online Banking (millions)	Total Number of Households (millions)
2002	27.3	109.3
2004	41.3	112.0
2006	58.6	114.4
2008	67.4	116.8
2010	72.5	117.5

- 7.3 Adding and Subtracting Polynomials 364
- 7.4 Multiplying Polynomials 372
- 7.5 Powers of Polynomials; Product of Binomial Conjugates 380
- 7.6 Properties of Exponents 386
- 7.7 Negative-Integer Exponents 393
- 7.8 Dividing Polynomials 403
- Taking it to the Lab: Climate Change Lab • Projectile Lab 409
- CHAPTER SUMMARY 411
- Key Points of Chapter 7 411
- Chapter 7 Review Exercises 414
- Chapter 7 Test 415

8

FACTORIZING POLYNOMIALS AND SOLVING POLYNOMIAL EQUATIONS 417

- 8.1 Factoring Trinomials of the Form $x^2 + bx + c$ and Differences of Two Squares 417
- 8.2 Factoring Out the GCF; Factoring by Grouping 424
- 8.3 Factoring Trinomials of the Form $ax^2 + bx + c$ 430
- 8.4 Sums and Differences of Cubes; A Factoring Strategy 437
- 8.5 Solving Polynomial Equations 441
- 8.6 Using Factoring to Make Predictions with Quadratic Models 450
- CHAPTER SUMMARY 458
- Key Points of Chapter 8 458
- Chapter 8 Review Exercises 460
- Chapter 8 Test 461
- Cumulative Review of Chapters 1–8 462

9

SOLVING QUADRATIC EQUATIONS 465

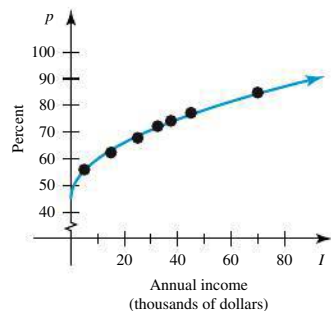
- 9.1 Simplifying Radical Expressions 465
- 9.2 Simplifying More Radical Expressions 471
- 9.3 Solving Quadratic Equations by the Square Root Property; The Pythagorean Theorem 477
- 9.4 Solving Quadratic Equations by Completing the Square 483
- 9.5 Solving Quadratic Equations by the Quadratic Formula 489
- 9.6 More Quadratic Models 497
- Taking it to the Lab: Climate Change Lab • Projectile Lab 506
- CHAPTER SUMMARY 507
- Key Points of Chapter 9 507
- Chapter 9 Review Exercises 509
- Chapter 9 Test 511

10

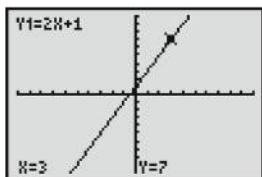
RATIONAL EXPRESSIONS AND EQUATIONS 513

- 10.1 Simplifying Rational Expressions 513
- 10.2 Multiplying and Dividing Rational Expressions; Converting Units 522
- 10.3 Adding Rational Expressions 529
- 10.4 Subtracting Rational Expressions 537

Adults Who Watch Cable
Television (pp. 597–598)



Tracing a Curve (p. 607)



- 10.5 Solving Rational Equations 543
- 10.6 Proportions; Similar Triangles 551
- 10.7 Variation 557
- 10.8 Simplifying Complex Rational Expressions 568
- Taking it to the Lab: Climate Change Lab • Estimating π Lab 575
- CHAPTER SUMMARY 576
- Key Points of Chapter 10 576
- Chapter 10 Review Exercises 579
- Chapter 10 Test 581

11

MORE RADICAL EXPRESSIONS AND EQUATIONS 582

- 11.1 Adding and Subtracting Radical Expressions 582
- 11.2 Multiplying Radical Expressions 586
- 11.3 Solving Square Root Equations 591
- CHAPTER SUMMARY 601
- Key Points of Chapter 11 601
- Chapter 11 Review Exercises 602
- Chapter 11 Test 603
- Cumulative Review of Chapters 1–11 603

A

USING A TI-83 OR TI-84 GRAPHING CALCULATOR 606

- A.1 Turning a Graphing Calculator On or Off 606
- A.2 Making the Screen Lighter or Darker 606
- A.3 Entering an Equation 607
- A.4 Graphing an Equation 607
- A.5 Tracing a Curve without a Scattergram 607
- A.6 Zooming 607
- A.7 Setting the Window Format 608
- A.8 Plotting Points in a Scattergram 609
- A.9 Tracing a Scattergram 609
- A.10 Graphing Equations with a Scattergram 610
- A.11 Tracing a Curve with a Scattergram 610
- A.12 Turning a Plotter On or Off 610
- A.13 Creating a Table 610
- A.14 Creating a Table for Two Equations 611
- A.15 Using “Ask” in a Table 611
- A.16 Plotting Points in Two Scattergrams 611
- A.17 Finding the Intersection Point(s) of Two Curves 612
- A.18 Finding the Minimum or Maximum of a Curve 612
- A.19 Finding any x -Intercepts of a Curve 613
- A.20 Turning an Equation On or Off 613
- A.21 Finding Coordinates of Points 613
- A.22 Entering an Equation by Using Y_n References 614
- A.23 Responding to Error Messages 614

ANSWERS TO ODD-NUMBERED EXERCISES 617

INDEX I-1

Preface

“The question of common sense is always, ‘What is it good for?’—a question which would abolish the rose and be answered triumphantly by the cabbage.”

—James Russell Lowell

These words seem to suggest that poet and editor James Russell Lowell (1819–1891) took elementary algebra. How many times have your students asked, “What is it good for?” After years of responding “You’ll find out in the next course,” I began a quest to develop a more satisfying and substantial response to my students’ query.

APPROACH AND ORGANIZATION

Curve-Fitting Approach Although there are many ways to center an elementary algebra course around authentic applications, I chose a curve-fitting approach for several reasons. A curve-fitting approach

- allows great flexibility in choosing interesting, authentic, current situations to model.
- emphasizes key concepts and skills in a natural, substantial way.
- deepens students’ understanding of equations in two variables, because it requires students to describe these equations graphically, numerically, symbolically, and verbally.
- unifies the many diverse topics of a typical elementary algebra course.

To fit a curve to data, students learn the following four-step modeling process:

1. Examine the data set to determine which type of model, if any, to use.
2. Find an equation of the model.
3. Verify that the model fits the data.
4. Use the model to make estimates and predictions.

This four-step process weaves together topics that are crucial to the course. Students must notice numerical patterns from data displayed in tables, recognize graphical patterns in scattergrams, find equations of models, graph models, and solve equations.

Not only does curve fitting foster cohesiveness within chapters, but it also creates a parallel theme for each set of chapters that introduces and discusses a new type of model. This structure enhances students’ abilities to observe similarities and differences among fundamental models such as linear models, quadratic models, rational models, and radical models.

Unique Organization Many college students who take elementary algebra had significant difficulties with the equivalent course in high school. These students face a greater challenge in the college course, because they must complete the course in one semester, rather than in two. Instead of presenting the material in the “same old way,” this text provides a unique organization that will better aid students in succeeding.

The text uses modeling to provide the “big picture” before going into details. For example, Chapter 1 gives an overview of linear modeling, which is the main theme of Chapters 1–6, and Section 7.2 provides an overview of quadratic modeling, which is a major focus of Chapters 7–9. Using modeling to provide the big picture not only is good pedagogy, but also sets the tone that this course will be different, interesting, alive, and relevant, inviting students’ creativity into the classroom.

The organization of the text is also unique in that students perform graphing (in Chapter 3) before they manipulate expressions and equations (in most of Chapters 4–11). That way, students can visualize the mathematics early on, making them better able to solve equations in one variable graphically. This early-graphing organization also buys

students a bit more time to find their “sea legs” before moving on to the more challenging manipulation work.

Several sections discuss how to use graphs and tables to solve equations in one variable.

NEW TO THE SECOND EDITION

Students will benefit from the following changes to the second edition of *Elementary Algebra: Graphs and Authentic Applications*:

- 149 data sets in examples and exercises have been replaced with more compelling and current topics.
- 175 data sets in examples and exercises have been augmented to include values for recent years.
- 55 new data sets have been added to address new concepts.
- All eight Climate Change labs have been updated.
- 100 conceptual exercises have been added.
- Percent applications have been added to Sections 2.6, 4.3, and 4.4.
- Sections 3.2 and 3.5 now contain graph-related modeling exercises.
- Additional examples and exercises addressing the commutative and associative laws have been added to Section 4.1.
- Modeling exercises that require students to define variables have been added to Sections 4.3, 4.4, 5.3, and 5.4.
- Authentic applications that do not involve curve fitting have been added to Sections 4.4, 5.3, 5.4, 7.6, and 7.7.
- The number of exercises addressing exponential properties has been increased in Sections 7.3 and 7.4.
- Section 7.8 on dividing polynomials has been added.
- Grids have been added to most graphs of models so students can better line up inputs and outputs.
- New Interactive Video Lecture Series are available in MyMathLab.
- The number of exercises available in MyMathLab/MathXL has been increased.

CONTINUED FROM THE FIRST EDITION

HOMEWORK SETS

The homework sets have been carefully structured so the exercises are well paired and progress gradually in difficulty. Students receive ample opportunities to master both procedures and concepts. In addition, two special types of exercises are included to help students succeed in elementary algebra and prepare them for their next course:

Related Review Exercises These exercises relate current concepts to previously learned concepts. The exercises occur near the end of all Homework sections in Chapters 5–11 and serve as constant reinforcement, helping students draw connections between previously learned and new material.

Expressions, Equations, and Graphs These exercises are designed to help students gain a solid understanding of those core concepts, including how to distinguish among them. The exercises are included at the end of all Homework sections in Chapters 5–11, but their foundation is laid in Section 4.5, which is devoted to making such distinctions.

BUILDING STUDY SKILLS

The following features have been included throughout the text to help students improve their study skills, to motivate students, and to provide just-in-time support.

Tips for Success Many sections close with tips for success in mathematics. These tips are intended to help students succeed in the course. A complete listing is included in the index.

Warnings These are discussions (flagged by the margin entry “WARNING”) that address students’ common misunderstandings about key concepts and help the students avoid such misunderstandings.

Chapter Opener Each chapter begins with a description of an authentic situation that can be modeled by the concepts discussed in the chapter.

TECHNOLOGY, EXPLORATIONS, AND LABS

Technology The text assumes students have access to technology, such as the TI-83 or TI-84 graphing calculator. Technology of this sort allows students to create scattergrams and check the fit of a model quickly and accurately. It also empowers students to verify their results from Homework exercises and efficiently explore mathematical concepts in the Group Explorations.

The text supports instructors in holding students accountable for all aspects of the course without the aid of technology, including finding equations of linear models. (Linear regression equations are included in the Answers section, because it can be difficult or impossible to anticipate which points a student will choose in trying to find a reasonable equation.)

Appendix A: Using a TI-83 or TI-84 Graphing Calculator Appendix A contains step-by-step instructions for using the TI-83 and TI-84 graphing calculators. A subset of this appendix can serve as a tutorial early in the course. In addition, when the text requires a new calculator skill, students are referred to the appropriate section in Appendix A.

Group Explorations All sections of the text contain one or two explorations that support student investigation of a concept. Instructors can use explorations as collaborative activities during class time or as part of homework assignments. Some explorations lead students to think about concepts introduced in the current section. Other, “Looking Ahead” explorations are directed-discovery activities that introduce key concepts to be discussed in the section that follows. The explorations empower students to become active explorers of mathematics and can open the door to the wonder and beauty of the subject.

Taking it to the Lab Laboratory assignments have been included at the end of most chapters, to increase students’ understanding of concepts and the scientific method. These labs reinforce the idea that mathematics is useful. They are also an excellent avenue for more in-depth writing assignments.

Some of the labs are about climate change and have been written at a higher reading level than the rest of the text in order to give students a sense of what it is like to perform research. Students will find that by carefully reading (and possibly rereading) the background information, they can comprehend the information and apply concepts they have learned in the course to make estimates and predictions about this compelling, current, and authentic situation.

RESOURCES FOR INSTRUCTORS

Instructor’s Resource Manual This manual contains suggestions for pacing the course and creating homework assignments. It discusses how to incorporate technology and how to structure lab and project assignments. The manual also contains section-by-section suggestions for presenting lectures and for undertaking the explorations in the text.

Instructor’s Solutions Manual This manual includes complete solutions to the even-numbered exercises in the homework sections of the text.

MyMathLab® Online Course (access code required) MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy to use format. MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit www.mymathlab.com or contact your Pearson representative.

New Ready to Go courses provide students with all the same great MyMathLab features, but make it easier for instructors to get started. Both the Standard and Ready To Go courses include pre-made homework and quizzes, which are pre-assigned in the Ready To Go course to make creating a course even easier.

TestGen TestGen enables instructors to build, edit, print, and administer tests by using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. Tests can be printed or administered online. The software and testbank are available for download from Pearson Education's online catalogue.

PowerPoint Lecture Slides (download only) Available through www.pearsonhighered.com or inside your MyMathLab course, these fully editable lecture slides include definitions, key concepts, and examples for use in a lecture setting and are available for each section of the text.

RESOURCES FOR STUDENTS

New Interactive Video Lecture Series This series has been completely revised to provide students with extra help for each section of the textbook. The Lecture Series includes:

- Interactive Lectures that highlight key examples and exercises for every section of the textbook.
- A new interface that allows easy navigation to sections, objectives, and examples.

These lectures are available in MyMathLab.

Student Solutions Manual This manual contains the complete solutions to the odd-numbered exercises in the Homework sections of the text.

GETTING IN TOUCH

I would love to hear from you and would greatly appreciate receiving your comments regarding this text. If you have any questions, please ask them, and I will respond.

Thank you for your interest in preserving the rose.

Jay Lehmann
MathNerdJay@aol.com

To the Student

You are about to embark on an exciting journey. In this course, you will not only learn more about algebra but also how to apply it to describe and make predictions about authentic situations. This text contains data that describe hundreds of situations. Most of the data have been collected from recent newspapers and Internet postings, so the information is current and of interest to the general public. I hope that includes you.

Working with authentic data will make mathematics more meaningful. While working with data about authentic situations, you will learn the meaning of mathematical concepts. As a result, the concepts will be easier to learn, because they will be connected to familiar contexts. And you will see that almost any situation can be viewed mathematically. That vision will help you understand the situation and make estimates and/or predictions.

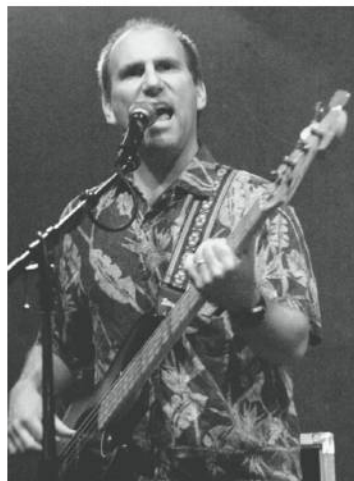
Many of the problems you will explore in this course involve data collected in a scientific experiment, survey, or census. The practical way to deal with such data sets is to use technology. So, a graphing calculator or computer system is required.

Hands-on explorations are rewarding and fun. This text contains explorations with step-by-step instructions that will lead you to *discover* concepts, rather than hear or read about them. Because discovering a concept is exciting, it is more likely to leave a lasting impression on you. Also, as you progress through the explorations, your ability to make intuitive leaps will improve, as will your confidence in doing mathematics. Over the years, students have remarked to me time and time again that they never dreamed that learning math could be so much fun.

This text contains special features to help you succeed. Many sections contain a Tips for Success feature. These tips are meant to inspire you to try new strategies to help you succeed in this course and future courses. Some tips might remind you of strategies you have used successfully in the past but have forgotten. If you browse through all of the tips early in the course, you can take advantage of as many of them as you wish. Then, as you progress through the text, you'll be reminded of your favorite strategies. A complete listing of Tips for Success is included in the index.

Other special features that are designed to support you in the course include Warnings, which can help you avoid common misunderstandings; Key Points summaries, which can help you review and retain concepts and skills addressed in the chapter you have just read; Related Review exercises, which can help you understand current concepts in the context of previously learned concepts; and Expressions, Equations, and Graphs exercises, which can help you understand and distinguish among these three core concepts.

Feel free to contact me. It is my pleasure to read and respond to e-mails from students who are using my text. If you have any questions or comments about the text, feel free to contact me through e-mail or the website www.pearsonhighered.com/lehmannseries.



Jay has a variety of interests. He is pictured here playing with his rock band, The Procrastinistas. (Photo courtesy of Rick Gilbert)

Jay Lehmann
MathNerdJay@aol.com

Acknowledgments

Writing a modeling textbook is an endurance run I could not have completed without the dedicated assistance of many people. First, I am greatly indebted to Keri, my wife, who yet again served as an irreplaceable sounding board for the multitude of decisions that went into creating this text. In particular, I credit her internal divining rod in selecting captivating data from a mound of data sets I have collected.

I have received much support from the following professors: Ken Brown, Gary Church, Jon Freedman, Eric Freidenreich, Jenny Freidenreich, Cheryl Gregory, Rick Hough, Evan Innerst, and Tadashi Tsuchida. Over the years, they have given much sound advice in responding to my countless e-mail inquiries.

I acknowledge several key people at Pearson Education. I'm very grateful to Editor-in-Chief Michael Hirsch, who has shared in my vision for this textbook and has made significant investments to make that vision happen. The book has been greatly enhanced through the support of Senior Acquisitions Editor Dawn Giovanniello, who has made a multitude of contributions, including assembling an incredible team to develop and produce this textbook. The team includes Senior Content Editor Lauren Morse, who handled countless tasks to support me in preparing the manuscript for production, leading to a significantly better book.

Heartfelt thanks goes to Integra's Associate Managing Editor Allison Campbell, who orchestrated the many aspects of production.

I also thank these reviewers, whose thoughtful, detailed comments helped me sculpt this text into its current form:

Scott Adamson, *Chandler-Gilbert Community College*
Thomas Adamson, *Phoenix College*
Ken Anderson, *Chemeketa Community College*
Mona Baarson, *Jackson Community College*
Sam Bazzi, *Henry Ford Community College*
Joel Berman, *Valencia Community College—East*
Ronnie Brown, *University of Baltimore*
Laurie Burton, *Western Oregon University*
James Cohen, *Los Medanos College*
Jeff Cohen, *El Camino College*
Joseph Deguzman, *Norco College*
Cynthia Ellis, *Purdue University, Fort Wayne*
Junko Forbes, *El Camino College*
Cathy Gardner, *Grand Valley State University*
James Gray, *Tacoma Community College*
Kathryn Gundersen, *Three Rivers Community College*
Miriam Harris-Botzum, *Lehigh Carbon Community College*
Tracey Hoy, *College of Lake County*
Evan Innerst, *Canada College*
Judy Kasabian, *El Camino College*
Charles Klein, *De Anza College*
Julianne M. Labbiento, *Lehigh Carbon Community College*
Jason Malozzi, *Lehigh Carbon Community College*
Debra Martin, *Purdue University, Fort Wayne*
Diane Mathios, *De Anza College*
Jim Matovina, *Community College of Southern Nevada*
Jane Mays, *Grand Valley State University*
Tim Merzenich, *Chemeketa Community College*
Jason Miner, *Santa Barbara City College*
Nolan Mitchell, *Chemeketa Community College*

Camille Moreno, *Cosumnes River College*
Ellen Musen, *Brookdale Community College*
Charlie Naffziger, *Central Oregon Community College*
Chris Nord, *Chemeketa Community College*
Karen D. Pain, *Palm Beach State College*
Ellen Rebold, *Brookdale Community College*
James Ryan, *State Center Community College District, Clovis*
Barbara Savage, *Roxbury Community College*
Ned Schillow, *Lehigh Carbon Community College*
David Shellabarger, *Lane Community College*
Laura Smallwood, *Chandler-Gilbert Community College*
Janet Teegarden, *Ivy Tech State College*
Cindy Vanderlaan, *Purdue University, Fort Wayne*
Lenove Vest, *Lower Columbia College*
Linda Wagner, *Purdue University, Fort Wayne*
Karen Wiechelman, *University of Louisiana at Lafayette*
Robin Williams, *Palomar College*

Index of Applications

A

acoustics

- examples of sound levels, 44
- sound inside running car, 402
- sound level of guitar, 398, 401, 566
- sound level of music from stereo system, 44
- sound produced by foghorn, 14, 15

aeronautics

- numbers of space debris, 52

age

- Americans ages 18–24 who smoked cigarettes, 506–7
- automobile accidents by, 23
- of automobiles
 - median, 42
 - value and, 34
- average annual expenditures by, 414
- of college, enrollment and, 254
- computer ownership and, 21
- drivers running red lights, 465, 503
- gray hair and, 21
- height and, 21
- home ownership and, 17–18, 50
- ideal, 270
- life expectancy and, 18–19
- median age of Harley-Davidson motorcycle buyers, 271
- percentage of American volunteers by, 21
- percentage of Americans who think they will live comfortably when retired, 280–81
- percentages of adults who approve of single men raising children on their own, 24
- percentages of Americans ordering takeout food, 23–24
- percentages of drivers stopped by police, 40–41

agriculture

- amounts of farmland in U.S., 262–63
- amounts of U.S. land used to grow organic cotton, 455
- annual consumption of cheese, 215
- changes in corn harvests, 87
- computers in farms, 505
- genetically modified crops, 599
- injuries on farms to people under age 20, 151
- number of cattle in the United States, 215
- number of farmers markets, 215

air travel. *See also* altitude

- Alaska Airlines ancillary revenues, 456
- average price per barrel of crude oil and total fuel cost for airline industry, 23
- claims filed against Transportation Security Administration (TSA), 348
- incline of climbing airplane, 141, 173, 239
- number of Americans who flew to Europe in 2007, 114
- numbers of firearms discovered at TSA airport checkpoints, 239
- percentage of airline seats filled in May, 261, 268
- percentage of flights delayed, 35–36
- percentages of on-time flights, 12
- revenue from fares, 233
- voluntary bumping rates of U.S. airlines, 43

altitude

- of airplane
 - change in, 113, 132
 - during descent, 20–21, 35, 132, 604
 - rate of change of, 154, 163
- of hot-air balloon, 25–26, 27, 30, 35

animals and pets

- deer population, 88
- dog years compared with human years, 261–62, 268–69, 339
- number of cattle in the United States, 215
- wolf population, 87

architecture

- base of building, 329, 476
- classroom size in school building design, 224
- room design, 329

area. *See also* geometry

- of rectangle, 3, 224, 230–31
- of trapezoid, 224, 231

art

- dimensions of frame for photograph, 230
- painting dimensions, 482
- painting's apparent height, 580–81

astronomy

- period of planet, 585
- weight on Jupiter, 555
- weight on Moon, 555

automobiles. *See also* motor vehicles

- accidents by age group, 23

ages of

- median, 42
- value and, 34

amount of gas in tank, 166

average selling prices for Subaru Outback, 166

decrease in Chrysler employees, 255–56

depreciation of, 215, 320

electric, 52

gas mileage, 50, 60, 130, 529

gasoline consumption and distances, 130, 164

gasoline consumption and driving time, 132, 166

gasoline–electric hybrid, 164, 283, 371, 499–501

GM's share of new-vehicle sales, 214, 360–61

Kia automobile sales per year, 233

market share of American automakers, 260

median ages of, 42

miles traveled and gallons of gasoline, 34, 164, 581

new-vehicle buyers who use Internet during shopping process, 232

Porsche 911 Carrera Turbo's fuel tank capacity, 10

Porsche 911 Turbo Coupe gas mileage, 529

privately owned vehicles entering the U.S., 344, 414–15

problems with, 151

seat belt usage in, 269–70

speed and stopping distance, 14, 15, 470–71, 566

speed limit and driving speed, 580

speed of, 2

total sales of gas, 231

Toyota Camry Hybrid gas consumption, 130

Toyota Prius and Camry Hybrid car sales, 87, 371

Toyota Prius gas consumption, 164

aviation. *See* air travel

B

banking. *See also* finance

average ATM fees, 229

checking account balances, 33, 60, 77–78, 173

households that use online, 513, 547–48

overdraft fees collected by banks, 9, 31, 75, 77

biology

- depth of visibility in Lake Tahoe, 270–71

DNA, electricity conduction in, 565

endangered or threatened species, 42

business. *See also* employment; profits; revenue; sales

advertising

- money spent by Google on advertising its products, 21
- price for 30-second ad during Academy Awards, 288

average life-spans of denominations of bills, 22

Bank of America revenue, 215

bicycle rental, 132, 556

changes in number of clients, 84

decrease in Chrysler employees, 255–56

depreciation of cars, 175, 320

employers giving workers paid day off for Martin Luther King, Jr. Day, 502

exports to U.S. by Costa Rica and Malaysia, 319

financial planner's client base, changes in, 84

international trade, 282–83, 319, 371

losses for various years, 12, 49

market share of American automakers, 260

market shares of Diet Pepsi and Coca-Cola Zero, 279

minimum wage for period 2009–2013, 128

number of farmers markets, 215

number of Quiznos Sub restaurants by year, 15

number of specialty bicycle shops in 2009, 107

number of years until inventions reach mass use, 24

numbers of McDonald's restaurants, 582, 600

office floor dimensions, 457

office space per worker, 215

percentage of Americans satisfied with size and power of major corporations, 22

percentages of Americans concerned about joblessness, 503–4

percentages of people working at age 65 or older, 502

revenue of Target, 163

revenues of Wal-Mart, 457–58

U.S. patent applications per year, 238

workplace illnesses and injuries, 505

- C**
- chemistry**
 acid solution, 327–28, 330, 349, 555
 alcohol solution, 326–27, 330
 antifreeze solution, 330
- communication.** *See also* telecommunications
 iPhone sales, 417, 456–57
 number of Americans who have cell phones, 10
 number of text messages sent or received, 107, 113
 number of wiretap authorizations, 362
 percentage of e-mail that is spam, 163
 percentage of households with wireless and no land-line telephones, 271
 percentages of teens who have cell phones, 44–45
 students who earned bachelor's degree in, 319
 telephone service costs, 555
 total cost for months of service, 231
 U.S. Postal Service first-class mail volume, 114
- computers**
 age and computer ownership, 21
 computer and video game sales, 10, 232
 cost of manufacturing, 60
 in farms, 505
 households with, income and, 262, 268
 Internet
 Americans who would eliminate accessing, 317–18
 households that use online banking, 513, 547–48
 households with broadband Internet access, 320, 549
 Internet gambling (e-gambling), 502
 media time Americans spent shopping online, 260
 new-vehicle buyers who use during shopping process, 232
 online crime complaints, 238
 percentage of Americans who have streamed a movie to their computer, 21
 percentage of public libraries with free access to e-books, 271
 percentage of public libraries with wireless Internet access, 12
 percentages of adult Internet users who use social networking sites, 117, 165, 372
 users in U.S., 117, 165
 U.S. households by income group who have access, 503
 online bill paying, 513
 online trading sites, 464
 percentage of e-mail that is spam, 163
 percentages of teenagers and adults who have watched online video, 456
 revenue of electronic gaming software, 107
 total value of goods sold on eBay, 505
 visitors to Facebook and Myspace, 320
- computer science, students who earned bachelor's degrees in,** 319
- construction, of telescopes,** 563–64
- consumer behavior**
 Americans considering home address to be personal information, 258–59, 264–65
 average annual expenditures by age group, 414
 household debt, 549
 households shopping at dollar stores, 281
 media time Americans spent shopping online, 260
 presents bought for holidays, 175
 purchasing decisions influenced by products made in the U.S., 23
 spending for Father's Day gifts, 164
 U.S. debit card purchases, 215
 water consumption per year, 580
- crime.** *See also* law and law enforcement
 California methamphetamine labs busted, 114
 firearm-related U.S. deaths, 263
 home security system sales, 271
 households burglarized in 2010, 581
 number of cocaine dealers arrested, 35
 number of complaints of online, 238
 numbers of assaults in New York City, 504
 numbers of justifiable homicides, 12
 percentages of drivers stopped by police, 40–41
 released convicts arrested for new, 600
 running red lights, 465, 503
 sexual harassment charges filed by men, 260
 violent-crime rate in U.S., 288
- criminology**
 DNA testing, 565
 prisoners and releases from prisons, 521, 550
- D**
- demographics**
 Americans ages 18–24 who smoked cigarettes, 506–7
 birthrate
 minorities in U.S., 271
 teenage, in U.S., 281, 378–79
 deaths
 firearm-related U.S., 263
 from heart disease, 378
 pedestrian, 174
 traffic fatality rates, 348
 foreign-born population, 362
 household sizes, 503
 life expectancy, 18–19
 living alone, 271, 319, 416
 married vs. never married 30–34-year-olds, 320
 sex ratios at birth in China, 363–64
- distance.** *See also* elevation
 across lake, 482
 between cities, 482–83
 driven in Metro rented truck, 264
 driven in U-Haul rented truck, 264
 driving in straight line from home to school, 482, 605
 driving in straight line from home to work, 482
 flying a kite and vertical to horizontal distance, 480–81
 to the horizon, 476
 rise of elevator over time, 60
 running time and, 169–70
 of sky diver fall, 392
 speed and, 132, 166
 steepness and vertical to horizontal distance
 comparing guy wires from ground to telephone pole, 134–35
 comparing ladders leaning against building, 143
 grade of road, 135, 141
 incline of climbing airplane, 141, 173, 239
 ski run decline, 141, 175, 604
 time traveled and, 33, 132
 at constant speed, 156–57, 176, 401
- E**
- economics**
 exports to U.S. by Costa Rica and Malaysia, 319
 international trade, 282–83, 319, 371
 median household incomes, 320
 percentages of Americans concerned about joblessness, 503–4
 personal incomes of Americans, 163, 364
- education**
 average cost of room and board at public universities, 458
 average salaries of public school teachers, 237, 378
 average starting salaries for employees with bachelor's degree, 24
 bachelor's degrees, 202, 319, 370
 earned by women and men, 297–98, 317
 business majors, 555
 calculating total course points, 70
 classroom floor area and number of students, 21, 224
 college
 cost of off-campus housing at Austin Community College, 269
 grade point average and acceptance in, 21
 number of two-year and four-year colleges, 89
 percentage full-time students, 555
 percentages of Americans 25 or older earning college degree, 261, 268
 percentages of freshmen with average high school grade of A, 177, 214
 student discretionary spending, 256–57
 students working full time, 49
 students working part time, 33
 units per semester, 10, 11, 520–21
 college enrollment
 age and, 34, 254
 of business majors, 555
 changes in, 80, 83, 87–88
 community college, 375–76
 in elementary algebra course, 262, 269
 fees in California, 164
 of female students, 59
 full-time equivalent (FTE), 98–99, 555
 in intermediate algebra course, 450–52
 of male students, 59
 rate of change in, 158
 of recent high school graduates, 518–19
 college tuition
 community college, 375–76
 fees and, 164
 in-state total expenses at Austin Community College, 269
 by number of credits, 21, 130, 132, 164, 559, 566, 581

- at public four-year vs. two-year colleges, 89
renting textbooks and, 132
total costs and, 59, 60
total student loan amounts, 280
enrollments at Thomas A. Edison High School, 457
high school dropout rate, 270
hours studying for quiz and score, 21
karate class costs, 155
law degrees earned by women, 502
vs. median annual income in 2001, 504–5
medical degrees earned by women, 511
National Assessment of Educational Progress (NAEP) test, 348
number of charter schools, 7–8
number of public school teachers, 378
part-time and full-time instructors, 294, 313–14, 359
percentage of women who were enrolled in school, 464–65
percentages of 10th graders who used marijuana in past 30 days, 361
percentages of elementary school students participating in after-school arts activity, 43–44
percentages of points on tests, 202, 239
students enrolled in algebra class, 10, 262, 269
time preparing for exam, 10
tutoring costs, 70, 167
U.S. school enrollments (kindergarten through grade 12), 357–58
- electricity**
conduction in DNA, 565
generated by nuclear power, 50
power in electrical circuit, 230
- electronics**
graphing calculator, using, 72–73
iPad sales, 10
percentage of Americans who own iPod vs. non-iPod portable MP3 player, 320
TI-84 Plus graphing calculator, cost of, 174
Xbox 360 4GB, cost of, 215
- elevation.** *See also* height
boiling point and, 159
from Dead Sea to Mount Everest, change in, 86
from Death Valley to Mount McKinley, change in, 86
depth of visibility in Lake Tahoe, 270–71
- height of towers on Golden Gate Bridge and, 82–83
- employment**
companies offering traditional benefit plans since 1990, 107
decrease in Chrysler employees, 255–56
employers giving workers paid day off for Martin Luther King, Jr. Day, 502
employers offering pension, 289
hours worked, 9
total amount of money earned and, 41, 556, 564
in week, 11
minimum wages increase in, 128
number of firefighters who died on duty, 165
number of public school teachers, 378
numbers of McDonald's restaurants, 582, 600
office space per worker, 215
over-65 workers, 502
participation rate, 318
percentage of workforce having manufacturing jobs, 264
percentage of youths employed/unemployed, 344
percentages of Americans concerned about joblessness, 503–4
salary
annual, 11
average hourly manufacturing pay, 23
average rate of change per year, 163, 215
average salaries for employees with education, 50
average salaries of public school teachers, 44, 237, 378
average salary for a hockey player, 214
average starting salaries for employees with bachelor's degree, 24
of baseball's Rodriguez, 52
bonuses, 53–54, 55
congressional pay, 106
hours worked and money earned, 60
years at company and, 21, 34, 52, 125–26, 131
union members, 10, 215
working from home, 2
- energy**
average price per barrel of crude oil and total fuel cost for airline industry, 23
burning old tires to create, 601
consumption of
by Asia and Oceania, 318
by North America, 318
petroleum, 371–72
by propane-gas barbecue grill, 164
cost of gasoline, 167
electricity generated by nuclear power, 50
gas mileage of car, 60
gasoline consumed after filling tank, 76, 264
gravitational potential, 230
onshore and offshore oil production, 605
- entertainment.** *See also* leisure and recreation; sports; television
Americans who would eliminate accessing Internet, 317–18
Americans who would eliminate watching television, 317–18
average movie ticket prices, 80
average ticket prices for concert tours, 1
drink cost and total cost, 130
fireworks used in U.S., 233
households that watched *Miss Universe Pageant*, 271
number of drive-in movie sites in U.S., 164
number of fans at rock concert, 10
number of people at club, 10
numbers of ride-related injuries at amusement parks, 45
percentages of Americans who go to movies, 43
per-person cost for party, 520
total box office gross from U.S. and Canada movie theaters, 49
Wye Oak concert ticket sales, 231
- environment**
depth of visibility in Lake Tahoe, 270–71
reused or recycled tires, 600–601
- F**
- farms.** *See* agriculture
- finance.** *See also* banking; investment
bicycle rental, 132, 556
checking account balance, 115, 173
college student discretionary spending, 256–57
costs
of buying pens, 21
of cable TV per subscription, 379
day-care, 174
of groceries, 555
of off-campus housing, 269
party, 167
per-person for party, 520
credit card balance, 73, 78, 94, 95–96, 99, 113, 116
currency exchange, 555
depreciation, 320
gasoline purchase, 528
home loan refinancing, 565
hourly wage, minimum, 128
household debt, 549
households that use online banking, 513, 547–48
income
average American household, 163, 320
education vs. median annual, 2001, 504–5
median sales prices of existing homes and, 89–90
number of U.S. billionaires, 50, 98
percentages put aside for savings, 198
personal, of Americans, 364
spent on food, 11, 44
unreported to the IRS, 585
U.S. household, collected by middle class, 288
interest
annual rate of, 329, 330, 346, 392, 402
earnings from, 232, 324–25, 329–30, 331, 392
from multiple accounts, 325–26, 329–30, 604
simple, 230, 232, 324
life-spans of denominations of bills, 22
lowering debt, 74
median price of home, 167
number of U.S. billionaires, 50, 98
online bill paying, 513
redistribute wealth by heavy taxes on rich, 555
savings account balance, 164, 175, 238, 239
savings over time, 254
for retirement, 163
service fees on checking account, 9, 75, 77
tax refunds, 457
total cost of parking plus dollars spent on a vase, 131
U.S. debit card purchases, 215
value of number of coins, 231
- firearms**
discovered at TSA airport checkpoints, 239
firearm-related deaths in U.S., 263
guns in circulation in United States, 43, 346
U.S. gun and ammunition revenues, 455
- food and nutrition**
Americans who support banning smoking in restaurants, 288

food and nutrition (*continued*)

annual consumption of
cheese, 215
annual sales of Echinacea, 271
average number of fast-food
meals consumed by young
adult, 163
butterfat in milk, 330
caffeine in Jolt, 529
cooking times of turkey in 325°
oven, 176
cost of pizza per person, 70
dividing pizza into slices, 61,
62, 65, 69
fair-trade coffee sales, growth
of, 132
fat in ravioli, 555
fish and shellfish
consumption, 528
genetically modified crops, 599
grocery costs per month, 555
income spent on, 70
lemon juice solution, 555
lime-juice solution, 326
meat consumption, 526–27
number of people participating
in Supplemental Nutrition
Assistance Program, 107
numbers of McDonald's
restaurants, 582, 600
ounces of diced tomatoes in
chicken cacciatore, 580
percentages of disposable
income spent on, 11
pizza weight, 392
price of bread, 60
recipe quantities, 98
salad dressing, 330
salt content of pretzels, 529
servings of sweetened
drinks, 202
sports drinks consumption, 12
sugar in cereal, 555
sugar in Coca-Cola, 529
suggested *vs.* actual baking
time for brownies, 131
takeout food ordering by age,
23–24
U.S. sales of soy energy
bars, 175
water consumed in year, 580
yogurt sales in United States,
159–60

G**games and contests**

Call of Duty video games, 239
revenue of electronic gaming
software, 107
shared lottery winnings, 41
worldwide sales of Sony
PlayStation 3, 600

garden

dimensions, 231, 329, 331, 457
enclosure, 238
width and area of, 225, 231

geometry

apparent height, 580–81

cube, volume of, 107
cylinder, height and volume of
water in, 47, 576
polygon, perimeter of, 224,
230, 237
rectangle
area of, 3, 11, 52, 58, 60, 70,
224, 230–31, 483
dimensions of, 329, 344, 457,
461, 464, 482
golden, dimensions of,
321–22, 329
length of diagonal, 482
perimeter of, 11, 49, 70, 113,
115, 225, 230–31, 239, 483
surface area of rectangular
box, 230
volume of rectangular
box, 392
sphere, volume of, 107
surface area of rectangular
box, 230
trapezoid, area of, 231
triangles
area of, 230
length of side of, 480, 482,
510, 511, 554, 556, 580,
581, 604
right, dimensions of, 480–81
similar, 553–54, 556, 580, 581
volume of cylinder, 392
volume of rectangular box, 231

global warming

average surface temperatures
of Earth, 46–47, 108,
168–69, 282
from carbon emissions, 108–10
burning fossil fuels and,
108, 282
China's population and, 506
in developing countries, 109,
282, 340–41, 506–7, 576
increases since 1950, 108,
109, 283, 506, 507, 575, 576
per-person carbon emissions,
283, 410, 506
U.S. population and, 410, 506
world population and, 409–10
Kyoto Protocol and, 109–10,
282–83
melting rate of Greenland's ice
cap, 168–69
population and, 283,
409–10, 506
from sources other than carbon
emissions, 108–9

government. *See also* politics

anti-government militia groups,
106–7
congressional pay, 106
Iraqi war death rates, 262, 268
number of African-American
federal and state
legislators, 260
number of overweight service
members, 162
number of special operation
soldiers, 271

number of suicides among
soldiers, 260
number of U.S. troop deaths in
Afghanistan War, 107
number of wiretap authoriza-
tions, 362
percentage of Americans who
are satisfied with size and
power of, 131
percentage of House seats by
region, 346
percentages of Americans
who said they have a
great deal of confidence
in Congress, 463
redistribute wealth by heavy
taxes on rich, 555

H**health**

adults who got flu shot, 602
AIDS deaths in U.S., 53, 83–84
Americans ages 18–24 who
smoked cigarettes, 505–7
Americans who support
banning smoking in
restaurants, 288
attention deficit hyperactivity
disorder (ADHD) in
boys, 458
blood donated to American
Red Cross, 215
death rates due to heart
disease, 43, 378
fast-food meals consumed by
young adult per year, 163
ideal weights
of men, 338–39
of women, 344–45
injuries on farms to people
under age 20, 151
Iraqi war death rates, 262, 268
life expectancy and, 18–19
lung cancer cases, 202
memory impairment, 510–11
mothers smoking cigarettes
during pregnancy, 271
number of firefighters who
died on duty, 165
number of health care fraud
prosecutions, 458
number of kidney
transplants, 288
number of living Americans
who have been diagnosed
with cancer, 202
number of overweight service
members, 162
numbers of antidepressant
prescriptions, 263
numbers of Lipitor and Crestor
prescriptions, 348
numbers of worldwide malaria
deaths, 461
nurse staffing and patient
mortality rates, 263, 270
ordinances that restrict
outdoor smoking, 260

pedestrian deaths per year, 174
percentages of Americans who
are uninsured, 52, 214
percentages of Americans
who have high
cholesterol, 503
percentages of obese American
adults, 51, 416
target heart-rate zone, 336–37
workplace illnesses and
injuries, 505
height. *See also* elevation
age and, 21
of baseball hit upward,
21, 457
of baseball thrown upward, 457
of cliff, 566
distance and, of painting,
580–81
of Freedom Tower *vs.* John
Hancock Tower, 98
of grass on golf putting
surfaces, 70–71
growth of a child, 58
of net on tennis court, 528
of object and length of its
shadow, 284–85
of person, 10
of stacked cups, 164
volume of water in cylinder *vs.*
height of cylinder, 47

I**insurance**

employee contribution
required to cover family
in employer-sponsored
health plan, 151
maximum income level to
qualify for NY Family
Health Plus, 163–64
offered to gay families, 163
percentage of adults insured
through employer *vs.*
government, 315–16
percentage of uninsured
Americans, 214

Internet. *See under* computers

investment. *See also* finance
current values and changes
in, 110
interest from, 324–25,
329, 330
percent change in, 110, 116
stock values, 34, 35
after bad publicity, 132
collecting data, 110
finding change in quantity, 79
last year *vs.* today, 87, 116
of Martha Stewart Living
Omnimedia, 214
number of shares and total
value, 59
from one month to the next,
115, 166
percent change of, 110, 264
since 2005, 104
total, 325

L**landscaping.** *See* garden**law and law enforcement.***See also* crime

border-patrol apprehensions, 262

home security system sales, 271

inmates younger than 18 held
in state prisons, 260number of health care fraud
prosecutions, 458

number of prisoners, 521, 550

number of wiretap

authorizations, 362

numbers of assaults in New

York City, 504

numbers of firearms
discovered at TSA airport
checkpoints, 239numbers of pages in the
IRS 1040 instruction
booklet, 363

ordinances that restrict

outdoor smoking, 260

percentage of Americans who
favor legalizing marijuana,
266–67**leisure and recreation.** *See also*

entertainment; sports

attendance at Broadway shows,
104–5

kite flying, 480–81

overnight visits to national
parks, 215

party costs, 167, 520

revenue of Broadway-based
movie musicals, 19–20

ski trip, 60, 517–18

worldwide music industry
revenues, 278**length**of 1974 Fender Jazz Bass,
526–27average length of side of
triangle, 230

of NBA basketball court, 528

of picture frame, 511

of rectangular carpet, 230

of rope, and number of knots
tied in it, 284of shadow, height of object and,
284–85

of telescopes, 563–64

lightamount, of light bulb,
402–3

brightness of, 566

illumination, 536

M**maps, scale of,** 555**media.** *See also* televisionaverage number of
magazine subscriptions
sold, 21average number of viewers of
local morning news on
television, 271

average time per day an

American spends reading

a daily newspaper, 260

daily newspapers, 260, 298,
317, 370numbers and revenues of print
newspapers, 580numbers of commercial and
educational FM radio
stations, 368–69

public radio listeners, 506

talk radio listenership, 360

time Americans spent shopping
online, 260**medicine**

adverse reactions to drugs, 239

blood donated to American
Red Cross, 215degrees in, earned by women,
150–51, 511

flu vaccine doses given, 602

numbers of antidepressant
prescriptions, 263percentages of Americans who
have high cholesterol, 503

radiation treatment, 563

sales of Viagra in U.S., 174

women obstetricians and
gynecologists, 280, 511**military**

defense spending, 21

Iraqi war death rates, 262, 268

number of overweight service
members, 162number of special operation
soldiers, 271number of suicides among
soldiers, 260numbers of troops in
Afghanistan national
army, 462percentages of Americans who
said they have a great deal
of confidence in, 414**miscellaneous**average release times for
balloon inflated with
single breath, 170–71households owning traditional
cameras, 281

ladder reach, 143, 482, 510

number of visits to U.S.

libraries, 237

number of visits to U.S.

slopes, 163

numbers of pages in the
IRS 1040 instruction
booklet, 363

waiter's tips, 202

word counts of Facebook's
privacy statements, 456**motor vehicles.** *See also*

automobiles

accidents

by age group, 23

decline in pedestrian

fatalities, 155

predicting fatality rate, 348

speeding male drivers in

fatal crashes, 605

years and number of U.S.
traffic deaths, 131

collision force, 555

gas mileage, 50, 60, 130, 529

gasoline tank fill rate, 164, 566

inside temperature, 555

Metro Truck Rental, 264

motorcycles

median age of buyers of

Harley-Davidson, 271

sales of, 116

percentages of drivers stopped

by police, 40–41

running red lights, 465, 503

stopping distance, 14, 15, 566

U-Haul Truck, 264

music

album purchases, 59, 505

CD collection

average rate of change per

year, 79

increases in, 79

concerts

average ticket prices for, 1,
36–39

revenue from, 107, 231

sale of CDs at, 30–31

total cost of tickets, 52, 237

cost of CDs, 231

guitar length, 526–27

guitar sound level, 398, 401, 566

guitar string vibration, 567

instrument sales, 208

instruments owned, 530–31

royalties paid to songwriters
and composers, 506

songs downloaded, 11, 163

songs learned by garage band
over time, 164sound levels played by
stereo, 44

studio recording time, 555

worldwide industry

revenues, 278

N**news.** *See also* mediaaverage number of viewers of
local morning news on
television, 271daily newspapers, 260, 298,
317, 370numbers and revenues of print
newspapers, 580**P****physics**behavior of airplane model in
wind tunnel, 27

force

of collision, 555

needed to lift object, 232

of spring, 566

on tool, 401, 566

intensity of radiation, 563

longwave radiation from

cloud, 565

period of pendulum, 585

period of planet, 585

power of windmill, 392

pressure added to bike tire, 132

projectile motion

ball, 411, 452–53

speed of projectile, 230

signal intensity, 392, 398, 566

tension of string, 562

vertical throwing speed, 411

vibration of guitar string, 567

volume of water in bathtub

after plug is pulled, 115

volume of water in cylinder vs.

height of cylinder, 47

volume of water pumped
out of flooded basement,
32, 114volumes and pressures in
syringe, 566–67**politics.** *See also* government

congressional pay, 106

percentages of Americans who
said they have a great
deal of confidence in
Congress, 463

presidential election voter

turnout, 86

private contributions to politi-
cal conventions, 113victorious gay candidates en-
dorsed by Gay & Lesbian
Victory Fund, 363women and men in House of
Representatives, 318–19**population**

deer, 88

of Gary, Indiana, since 1980, 107

land areas for various states
and, 99sex ratios at birth in China,
363–64

Steller's sea lion, 163

United States, 410

wolf, 87

profits

annual, 10, 12, 34, 49

from CD sales, 30–31

company, 457, 464

of a company since 2005, 51, 85

publishing

author earnings from book

agent, 202

numbers of pages in books of

Harry Potter series, 22

numbers of pages in the

IRS 1040 instruction
booklet, 363

royalties, 580

R**real estate**age and home ownership,
17–18, 50average selling prices of
homes, 215

real estate (*continued*)

home security systems, 271
 homes occupied by owners, 512
 house rental, 56
 income spent on mortgages, 70
 median sales prices of homes and incomes, 89–90
 number of householders owning home, 17
 number of people who live in two-bedroom house, 2
 office space per worker, 215
 people living in two-bedroom house, 2
 value of new home, 11

religion

Americans regularly attending church in typical week, 605
 Americans who believe humans evolved without God, 271
 importance of, to Americans, 271

revenue

from ADHD drugs, 50
 from air fares, 233
 Alaska Airlines ancillary, 456
 annual, 35
 of Apple, 12
 from balcony seat sales, 323–24, 329
 Bank of America, 215
 of Broadway-based movie musicals, 19–20
 from Broadway shows, 19–20
 company, 457, 462
 from downloaded album sales, 505
 of electronic gaming software, 107
 from gambling, 456
 from hot dog and hamburger sales, 322–23
 from *The Hunger Games* book and CD sales, 329
 increases by year, 164
 from Internet gambling (e-gambling), 502
 from The Kills album and EP sales, 329
 from music concerts, 107
 music industry, worldwide, 278
 from organic-food sales, 145–46
 from personal breathalyzers, 107
 from Radiohead album sales, 329
 from royalties, 506, 580
 from sales of soy energy bars, 175
 from selling Dr. Grip pens, 605
 of Starbucks, 115
 of sugar-free/reduced-sugar foods, 42, 458
 of Target, 163
 from televisions, 107
 from ticket sales, 329, 330, 344, 346, 463

U.S. gun and ammunition, 455
 of Viagra in U.S., 174
 of Wal-Mart, 457–58
 from *You Killed Wesley Payne* book sales, 329

S

sales. *See also* profits; revenue
 of albums, 59, 132
 books, 202
 for *Call of Duty* video games, 239
 camera, 208
 of CDs at concerts, 30–31
 of computer and video games, 10, 232
 concert ticket prices and, 1, 36–39
 of downloaded albums, 505
 on eBay, 505
 Echinacea, 271
 fair-trade coffee, 132
 gasoline–electric hybrid cars, 499–501
 GM's share of new-vehicle sales, 214
 of home security systems, 271
 Honda Accord, changes in, 320
 of hot dogs, 54, 55, 322–23
 of iPads, 10
 of iPhones, 417, 456–57
 Kia automobiles per year, 233
 of LCD televisions, 78
 of loaves of bread, 60
 of magazine subscriptions by telemarketer, 21
 market shares of Diet Pepsi and Coca-Cola Zero, 279
 of motorcycles, 116
 music, 360
 of musical instruments, 208
 numbers of years and, 131
 prices
 of digital televisions, 361–62
 retail price and net price, 130
 royalties and, 506, 580
 of socks, 59
 of Sony PlayStation 3, 600
 of soy energy bars in U.S., 175
 total gasoline sales, 231
 total value of goods sold on eBay, 505
 of Toyota Camry Hybrid, 371
 of Toyota Prius, 87, 371
 of Viagra in U.S., 174
 of yogurt in United States, 159–60
society. *See also* demographics
 Americans who support banning smoking in restaurants, 288
 approval of single men raising children on their own, 24
 married-couple households, 280
 married people reaching major anniversaries, 361

married vs. never married
 30–34-year-olds, 320
 number of Americans living alone, 271, 319, 416
 percentage of Americans who believe gay/lesbian relations are morally acceptable, 207
 percentage of Americans who favor legalizing marijuana, 266–67
 percentage of Americans who think they will live comfortably when retired, 280–81
 percentages of Americans concerned about joblessness, 503–4
 percentages of married persons very happy with their marriages, 240, 269

sound. *See* acoustics

speed

airplane, 166
 commuting time and, 552
 driving, 59, 528, 580
 driving times and distance, 54, 60, 132, 156–57, 166, 233, 536
 land speed records, 263
 of projectile, 230
 runner's pulse rate and, 15
 running, 169–70, 176
 speed limit in miles per hour, 59, 60, 581
 stopping distance and, 14, 15, 470–71, 566
 vertical throwing speed, 411
 walking, 169–70, 526–27

sports

baseball
 attendance, vs. football, 98
 career grand slams, 25
 height of ball hit upward, 21
 Mays' stolen bases by year, 51
 Miami Marlins' and New York Mets' payroll, 320
 number of home runs, 10
 number of Little League baseball participants, 237
 salary of Rodriguez, 52
 steroid use and Hall of Fame eligibility, 553
 ticket prices, 114, 225–26, 231
 basketball
 Americans who have ever played organized, 504
 foreign-born players on NCAA Division I men's teams, 585
 hoop height, 526–27
 NBA court length, 528
 college, core GPAs/SAT scores needed to qualify to play, 165–66
 football
 average attendance at college bowl games, 264

average Super Bowl ticket prices, 16–17
 football field, 329, 482
 total cost of differently priced tickets, 231
 golf
 grass heights on golf putting surfaces, 70–71
 Woods's golf tournament earnings and numbers of wins, 22–23
 hockey
 average salary for player, 214
 New York City Marathon numbers of women who have run in, 504
 Olympics
 100-meter men's freestyle winning times, 259
 500-meter speed-skating winning times, 298, 317
 restricted to amateur athletes, 555
 percentage of children in organized, 10
 running
 10-kilometer run, 528
 200-meter run world record times, 290, 298, 317
 800-meter run, 314–15
 runner's pulse rate and speed, 15
 soccer
 percentage of American adults interested in, 362, 604–5
 soccer field, 482
 softball league, cost of joining, 114
 table tennis (ping pong) table, 329
 tennis
 height of net, 528
 prize money for women's singles Wimbledon champion, 12
 tennis court, 329
 volleyball court, 230

survey

of car buyers for average number of vehicle problems, 151
 percentage of Americans who favor legalizing marijuana, 266–67

T**taxes**

redistribute wealth by heavy taxes on rich, 555
 sales, in Chicago, 201
 tax audit rates, 114
 total costs to individuals and companies to prepare, 23

telecommunications

Internet. *See under* computers.

- percentage of households with wireless and no land-line telephones, 271
- total cost for months of service, 231, 555
- television.** *See also* media
- ad-supported broadcast TV, time spent watching, 318
 - Americans who would eliminate watching, 317–18
 - average number of viewers of local morning news on, 271
 - average prices of digital televisions, 361–62
 - broadcast TV, 318
 - cable
 - ad-supported, time spent watching, 318
 - basic rates for, 115, 379, 555
 - households with, 521, 550
 - percentage of American adults watching, income and, 597–98
 - dimensions of screen, 471, 482
 - households that watched *Miss Universe Pageant*, 271
 - numbers of TV viewers of *Good Morning America* vs. *Early Show*, 98
 - price for 30-second ad during Academy Awards, 288
 - prime-time television, percentages of Americans who watched, 497–99
 - revenue from, 107
 - satellite TV subscriptions, 202
 - signal intensity, 392, 398, 566
 - viewers who watch programs within specified number of days, 603
 - worldwide shipments of LCD and plasma televisions, 371
- temperature,** 10. *See also* global warming
- boiling point and elevation, 159
 - change in
 - calculating increase/decrease, 462, 604
 - rate of change, 154–55, 163, 166, 174
 - cooling rate of potato since removed from oven, 21
 - decrease in, over time, 78, 86, 113, 114, 115
 - Fahrenheit vs. Celsius, 164–65, 226–29
 - increase in, over time, 75, 78, 81–82, 83, 86, 88, 154
 - inside car, 555
 - longwave radiation from cloud and, 565
 - low
 - for four days in December in Indianapolis, 52
 - for four days in March in New York City, 346
 - for three days in December in Chicago, 12
 - “Murphy’s law” of, 565
 - in oven, 11
 - rate a cricket chirps and, 346
 - relative humidity and, 565
- tests and testing**
- average response time, 58
 - National Assessment of Educational Progress (NAEP), 348
 - practice exams, 41
 - scores
 - average, 12, 59, 232
 - average SAT, 59, 60
 - bonus points added to, 41
 - quiz scores, 6–7, 13–14, 15, 20
 - on second exam and changes in scores, 87, 239
 - time preparing for exam, 10
- time**
- number of years since specific date, 2, 10, 18, 20, 37, 45, 49, 60, 208, 267, 279, 370, 375–76, 378–79, 414, 416, 600
- trade, international**
- exports to U.S. by Costa Rica and Malaysia, 319
 - worldwide shipments of LCD and plasma televisions, 371
- transportation.** *See also* air travel;
- motor vehicles
 - taxi charges, 174, 215
 - trips taken by Americans on public transportation, 215
- travel.** *See also* air travel; motor vehicles
- annual lost time due to traffic congestion/delays, 12, 165
 - average times spent driving daily, 349, 355–56
 - commuting time, 552
 - driving time
 - gasoline consumption and, 132, 164, 264
 - speed and, 132, 536
 - miles traveled and gallons of gasoline, 164
 - miles traveled per week, 552
 - number of visits to U.S. slopes, 163
 - overnight visits to national parks, 215
 - seat belt usage, 269–70
 - ski trip cost, 60, 517–18
 - trips taken by Americans on public transportation, 215
 - websites, 22
- V**
- volunteer work**
- blood donated to American Red Cross, 215
 - percentage of American volunteers by age, 21
- W**
- weather**
- relative humidity, 565
- weight**
- of baby at birth, 2
 - before and after diet, 78, 127, 174
 - of fish/shellfish consumed by Americans in 2011, 528
 - ideal
 - of men, 338–39
 - of women, 344–45
 - on Jupiter, 555
 - loss program, 174, 269
 - of math professor, 269
 - of meat consumed by Americans in 2011, 526–27
 - on Moon, 555
 - of pizza, 392, 566
 - of service members, 162
 - of textbooks, 566
 - of TI-84 graphing calculator, 580
- work.** *See* employment; volunteer work

Introduction to Modeling



Think about the last concert you attended. What was the ticket price? Was it worth it? The average ticket price for the top-50-grossing concert tours has increased greatly (see Table 1). In Example 2 of Section 1.4, we will predict when the average ticket price will be \$105.

In this course, we will discuss how to describe the relationship between two quantities that occur in an authentic situation. For example, we will describe how the average ticket price for the top-50-grossing concert tours has changed over time. In Chapters 1–6, we will focus on how to use (straight) lines to describe authentic situations. In Chapters 7–11, we will discuss other types of *curves* that can be used to describe authentic situations.

Table 1 Average Ticket Prices for Top-50-Grossing Concert Tours

Year	Average Ticket Price (dollars)
1998	33
2001	47
2004	59
2008	67
2011	85

Source: *Pollstar*

1.1 Variables and Constants

Objectives

- » Know the meaning of *variable* and *constant*.
- » Know the meaning of *counting numbers*, *integers*, *rational numbers*, *irrational numbers*, *real numbers*, *positive numbers*, and *negative numbers*.
- » Use a number line to describe numbers.
- » Graph data.
- » Find the average (or mean) of a group of numbers.
- » Know how to describe a concept or procedure.

In this section, we will work with *variables* and *constants*, two extremely important building blocks of algebra. We will also discuss various types of numbers and how to describe numbers visually.

Variables

In arithmetic, we work with numbers. In algebra, we work with *variables* as well as numbers.

Definition Variable

A **variable** is a symbol that represents a quantity that can vary.

For example, we can define h to be the height (in feet) of a specific child. Height is a quantity that varies: As time passes, the child's height will increase. So, h is a variable. When we say $h = 4$, we mean the child's height is 4 feet.

We will discuss other roles of a variable in Sections 2.1 and 4.3.



Example 1 Using a Variable to Represent a Quantity

1. Let s be a car's speed (in miles per hour). What is the meaning of $s = 60$?
2. Let n be the number of people (in millions) who work from home at least once a week during normal business hours. For the year 2010, $n = 22$ (Source: *World at Work*). What does that mean in this situation?
3. Let t be the number of years since 2010. What is the meaning of $t = 4$?

Solution

1. The speed of the car is 60 miles per hour.
2. In 2010, 22 million people worked from home at least once a week during normal business hours.
3. $2010 + 4 = 2014$; so, $t = 4$ represents the year 2014.

There are many benefits to using variables. For example, in Problem 2 of Example 1, we found that the simple equation " $n = 22$ " means the same thing as the wordy sentence "22 million people worked from home at least once a week during normal business hours." Variables can help us describe some situations with a small amount of writing.

In Problem 3 of Example 1, we described the year 2014 by using $t = 4$. So, our definition of t allows us to use smaller numbers to describe various years—an approach that will be helpful throughout the course.

We will see other benefits of variables as we proceed through the course.

Example 2 Using a Variable to Represent a Quantity

Choose a symbol to represent the given quantity. Explain why the symbol is a variable. Give two numbers that the variable can represent and two numbers that it cannot represent.

1. the weight (in pounds) of a baby at birth
2. the number of people who live in a two-bedroom house

Solution

1. Let w be the weight (in pounds) of a baby at birth. The weight of a baby at birth can vary, so w is a variable. For example, w can represent the numbers 6 and 8, because babies can weigh 6 or 8 pounds at birth. The variable w does not represent 0 or 300, because babies cannot weigh 0 or 300 pounds at birth!
2. Let n be the number of people who live in a two-bedroom house. The number of people who live in a two-bedroom house can vary, so n is a variable. For example, n can represent the numbers 2 and 3, because 2 or 3 people can live in a two-bedroom house. The variable n cannot represent the numbers 5000 or $\frac{1}{2}$, because 5000 people cannot live in a two-bedroom house and half of a person doesn't make sense.

In Problem 1 of Example 2, we stated that the units of w are pounds. Without stating the units of w , " $w = 10$ " could mean the baby's weight was 10 ounces, 10 pounds, or 10 tons! In defining a variable, it is important to describe the variable's units.

Constants

A variable is a symbol that represents a quantity that can vary. When we use a symbol to represent a quantity that does *not* vary, we call that symbol a *constant*. So, 2, 0, 4.8, and π are constants. The constant π is approximately equal to 3.14.

Definition Constant

A **constant** is a symbol that represents a specific number (a quantity that does *not* vary).



Figure 1 One square inch

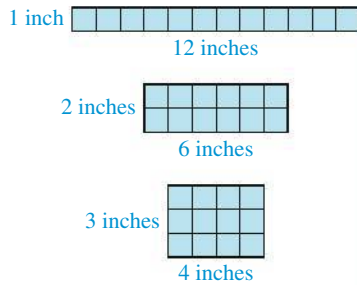


Figure 2 Three possible rectangles of area 12 square inches

In the next example, we will compare the meanings of a variable and a constant while we consider the widths, lengths, and areas of some rectangles. The **area** (in square inches) of a flat surface is the number of square inches that it takes to cover the surface (see Fig. 1). The area of a rectangle is equal to the rectangle's length times its width.

Example 3 Comparing Constants and Variables

A rectangle has an area of 12 square inches. Let W be the width (in inches), L be the length (in inches), and A be the area (in square inches).

1. Sketch three possible rectangles of area 12 square inches.
2. Which of the symbols W , L , and A are variables? Explain.
3. Which of the symbols W , L , and A are constants? Explain.

Solution

1. We sketch three rectangles for which the width times the length is equal to 12 square inches (see Fig. 2).
2. The symbols W and L are variables, since they represent quantities that vary.
3. The symbol A is a constant, because in this problem the area does not vary—the area is always 12 square inches.

Counting Numbers

When we describe people, it often helps to describe them in terms of certain categories, such as gender, ethnicity, and employment. In mathematics, it helps to describe numbers in terms of categories, too. We begin by describing the *counting numbers*, which are the numbers 1, 2, 3, 4, 5, and so on.

Definition Counting numbers (natural numbers)

The **counting numbers**, or **natural numbers**, are the numbers

$$1, 2, 3, 4, 5, \dots$$

The three dots mean that the pattern of the numbers shown continues without ending. In this case, the pattern continues with 6, 7, 8, and so on. When a list of numbers goes on forever, we say that there are an *infinite* number of numbers.

Integers

Next, we describe the *integers*, which include the counting numbers and other numbers.

Definition Integers

The **integers** are the numbers

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

The three dots on both sides mean that the pattern of the numbers shown continues without ending in both directions. In this case, the pattern continues with -4 , -5 , -6 , and so on, and with 4, 5, 6, and so on.

The **positive integers** are the numbers 1, 2, 3, \dots . The **negative integers** are the numbers -1 , -2 , -3 , \dots . The integer 0 is neither positive nor negative. So, the integers consist of the counting numbers (which are positive integers), the negative integers, and 0.

The Number Line

We can visualize numbers on a *number line* (see Fig. 3).

Each point (location) on the number line represents a number. The numbers increase from left to right. We refer to the distance between two consecutive integers on the number line as 1 *unit* (see Fig. 3).

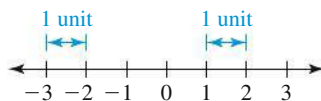


Figure 3 The number line